Assignment no-5

(1)$\lim_{x\to 0}\left(1+\frac{3x}{2}\right)^{1/x}$

(2) $\lim_{x\to 0}\left(\frac{1-x}{1+x}\right)^{1/x}$

(3)$\lim_{x\to 0}\left(\frac{1+7x}{1-5x}\right)^{1/x}$

(4) $\lim_{x\to 0}\left(\frac{4x+1}{1-4x}\right)^{1/x}$

(5)$\lim_{x\to 0}\left(\frac{2+x}{2-x}\right)^{1/x}$

(6)$\lim_{x\to 0}\left(\frac{3+2x}{3-x}\right)^{1/x}$

(7)$\lim_{x\to 0}\left(\frac{5+2x}{5-3x}\right)^{1/x}$

(8)$\lim_{y\to 3}(y-2)^{2/(3y-9)}$

(9)$\lim_{x\to 2}(x-1)^{1/x-2}$

(10)$\lim_{y\to 5}(y-4)^{2/(3y-15)}$

Answer : (1)$e^{3/2}$ (2)e^{-2} (3)e^{12} (4)e^{8} (5)e (6)e (7)e (8)$e^{2/3}$ (9)e (10)$e^{2/3}$.

Assignment No-6

(1)$\lim_{x\to 0}\frac{\log(1+5x)}{x}$

(2)$\lim_{x\to 0}\frac{log5+\log\left(x+\frac{1}{5}\right)}{x}$

(3)$\lim_{x\to 2}\frac{\log x-\log 2}{x-2}$

(4)$\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}$

(5)$\lim_{x\to 0}\frac{\log(7+x)-\log(7-x)}{\tan x}$

(6)$\lim_{x\to 7}\frac{\log x-\log 7}{x^2-49}$

(7)$\lim_{x\to 0}\frac{\log(1-7x)}{\sin 5x}$

(8)$\lim_{x\to 1}\frac{\log x}{x-1}$

(9)$\lim_{x\to 0}\frac{\log(1+x)}{3^x-1}$

(10)$\lim_{x\to 0}\frac{(3^{sinx}-1)^2}{x.\log(1+x)}$

(11)$\lim_{x\to 0}\frac{\log(a+x)-\log(a-x)}{x}$

(12)$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{\log(1+x)}$

(13)$\lim_{x\to 0}\frac{(3^x-1)^3}{(2^x-1).\sin x.\log(1+x)}$

(14)$\lim_{x\to \pi/2}\left(\frac{1+\cot x}{1+\cos x}\right)^{1/\cos x}$

(15)$\lim_{x\to 0}\frac{(e^{2x}-1)(1-cosx)}{tan^2x.\log(1+2x)}$

Answer : (1) 5 (2) 1 (3)1/2 (4)2/3 (5)2/7 (6)1/68 (7) -7/5 (8)1 (9)1/log 3 (10) $(\log 3)^2$ (11)2/a (12)$\frac{1}{2}$ (13)$\frac{(\log 3)^2}{log2}$ (14)1 (15)1/2

Assignment No -7

(1)$\lim_{x\to`\infty}\frac{3x^2+4x+4}{4x^2+7x+8}$

(2)$\lim_{x\to\infty}\frac{x^3-3x^2+5x+6}{4x^3+x^2-x-1}$

(3)$\lim_{x\to\infty}\frac{(3x+4)(4x-6)(5x+2)}{4x^3+2x^2-1}$

(4)$\lim_{x\to\infty}\frac{(2x-3)^2.(3x+2)^3}{(2x+1)^5}$

(5)$\lim_{x\to\infty}\frac{x^3-6}{\sqrt{9+4x^6}}$

(6)$\lim_{x\to\infty}\frac{\sqrt{16x^2+1}-\sqrt{9x^2+4}}{3x+5}$

(7)$\lim_{x\to\infty}\frac{\sqrt{3x^2+4x+5}-\sqrt{2x^2+3}}{3x+7}$

(8) $\lim_{x\to\infty}\frac{x+4}{\sqrt{x^2+1}+\sqrt{x^2-5}}$

(9)$\lim_{x\to\infty}(\sqrt{x^2+3x}-x$

(10)$\lim_{x\to\infty}\sqrt{x+\sqrt{x}}-\sqrt{x}$

(11)$\lim_{x\to\infty}x(\sqrt{x^2+1}-\sqrt{x^2-1})$

(12)$\lim_{x\to\infty}\sqrt{3x}\,(\sqrt{x+1}-\sqrt{x})$

(13)$\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x})(\sqrt{x+\frac{1}{2}})$

(14)$\lim_{x\to\infty}x\sin(\frac{1}{x})$

(15) $\lim_{x\to\infty}x^3\sin(\frac{1}{x^4})$

Answers : (1)3/4 (2)1/4 (3)15 (4)27/8 (5)1/2 (6)1/3 (7)$\frac{\sqrt{3}-\sqrt{2}}{3}$ (8)1/2 (9)3/2 (10)1/2 (11)1 (12)$\sqrt{3}$/2 (13)1/2 (14)1 (15)0.

Assigment No -8

(1)$\lim_{x\to0}\frac{\cos x-\sqrt{\cos x}}{\sin^2 x}$

(2)$\lim_{x\to0}\frac{\tan x-\sin x}{\sin 3x-3\sin x}$

(3)$\lim_{\theta\to0}\frac{1-\cos m\theta}{1-\cos n\theta}$

(4)$\lim_{x\to0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

(5)$\lim_{x\to0}\frac{\cos(a+x)+\cos(a-x)-2\cos a}{x\tan x}$

(6)$\lim_{x\to\pi/4}\frac{(\sin x-\cos x)^2}{\sqrt{2}-\sin x-\cos x}$

(7)$\lim_{x\to\pi}\frac{\sqrt{5+\cos x}-2}{(\pi-x)^2}$

(8)$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}$

(9)$\lim_{x\to\pi/6}\frac{2\sin x-1}{\pi-6x}$

(10)$\lim_{x\to a}\frac{\sqrt{\cot x}-\sqrt{\cot a}}{x-a}$

(11)$\lim_{x\to0}\frac{e^{6x}-e^{4x}-e^{2x}+1}{x\tan x}$

(12)$\lim_{x\to0}\frac{(e^{2x}-1)(1-\cos 2x)}{x^3}$

(13)$\lim_{x\to0}\frac{(9^x-1)(1-\cos x)}{x^3}$

(14)$\lim_{x\to0}\frac{(4^x-1)(1-\cos 4x)}{3\sin x-\sin 3x}$

(15)$\lim_{x\to0}\frac{(6^x-1)^2}{x\log(1+2x)}$

(16)$\lim_{x\to\pi/4}(1+\cos 2x)^{4\sec 2x}$

(17)$\lim_{x\to a}(\frac{\sin x}{\sin a})^{\frac{1}{x-a}}$

(18)$\lim_{x\to0}\frac{(5^x-1)^3}{(3^x-1).\sin 5x.\log(1+x)}$

(19)$\lim_{x\to2}\frac{x-2}{\log_a(x-1)}$

(20)$\lim_{x\to\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$

(21)$\lim_{x\to\infty}\frac{4x^2+x-1}{\sqrt{x^4+5}+2}$

(22)$\lim_{x\to\infty}\frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}}$

(23)$\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x})(\sqrt{x+\frac{1}{4}})$

(24)$\lim_{n\to\infty}(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots+\frac{1}{n})$

(25)$\lim_{x\to\infty}x^4.\sin(\frac{2}{x^5})$

Answers : (1)-1/4 (2)-1/8 (3)m^2/n^2 (4)3/2 (5) – cos a (6)$2\sqrt{2}$ (7)1/8 (8)$1/16\sqrt{2}$

(9)- $\sqrt{3}$/6 (10)$\frac{-\,cosec^2\,a}{2\sqrt{\cot\,a}}$ (11)8 (12)4

(13) log 3 (14) 4 log 2 (15)$\frac{1}{2}(\log 6)^2$

(16)e^4 (17)$e^{\cot a}$ (18)$\frac{(\log 5)^3}{5\log 3}$ (19)$\log a$

(20)1 (21)4 (22)$\frac{a^2-b^2}{c^2-d^2}$ (23)1/2 (24)1/2

(25)0.

Assignment no -9

Discuss the continuity of the function f givenby

(1)F(x) =$\frac{x^2-64}{x^2-16}$, when x≠ 4

= 12 when x = 4 at x=4

(2)F(x) =$\frac{x^2-4x+3}{x^2-1}$, if x≠ 1

= -1, if x = 1 at x = 1.

(3) F (x) = $\frac{\sqrt{x+6}-3}{x^2-9}$, for x ≠ 3

=1/2, for x =3 at x =3

(4) F(x) = $\frac{\sqrt{4+x}-2}{x}$, for x ≠ 0

= 0, for x = 0 at x = 0**(M '03)**

(5) F(x) = $\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$, for x ≠ 2

= 1 / 2 for x = 2 at x = 2.

(Oct. '04)

(6) F(x) = $\frac{\tan(x^2-x)}{x}$, for x ≠ 0

=2, for x = 0 at x = 0 .

(7) F(x) = $\frac{5^x-e^x}{sin2x}$, for x ≠ 0

=$\frac{1}{2}$ (log 5-1), for x = 0 at x = o.

(8) F(x) = $\frac{3^x+3^{-x}-2}{x^2}$, for x ≠ 0

=$(\log 3)^2$ for x = 0 at x = 0

(9) F(x) = $\frac{(e^{2x}-1)\sin x}{x^2}$, if x ≠ 0

= 4, if x = 0 at x = 0

(10) F(x) = $\frac{\log x-\log 3}{x^2-9}$, for x ≠ 3

= $\frac{1}{18}$, for x =3 at x = 3.

(11) F(x) = $\frac{x-1}{\log x}$, for x ≠ 1

=1, for x =1 at x = 1

(12) F(x) =$\frac{5^{cosx}-1}{\frac{\pi}{2}-x}$, for x ≠ **π** / 2

= 2 log 5 , for x = **π** /2 at x =$\frac{\pi}{2}$.

Answers :

Continuous : 2,7,8,10,11.

Discontinuous :1,3,4,5,6,9,12.

Assignment no . 10

Show that each of the following functions have removable discontinuity at the point shown. Against it. Remove this discontinuity by redefining the function suitably .

(1)F(x) = $\frac{x^2-9}{x-3}$, for x ≠ 3

= 4 , for x =3 at x = 3.

(2) F (x) = x^2 sin($\frac{1}{x}$) For x ≠ 0

=1, For x = 0 at x = 0.

(3)F (x) = $(\sqrt{\frac{4x+1}{1-4x}})^{1/x}$, For x ≠ 0

=e^8 For x = 0 at x = 0.

(4) F(x) = $\frac{(e^{3x}-1)\sin x}{x^2}$, For x ≠ 0

=4, For x = 0 at x = 0.

(5) F (x) = $\frac{(5^x-1)^2 \cdot x}{\tan^2 x \cdot \log(1+x)}$, it x ≠ 0

= 2 log 5, x = 0 at x = 0.

(6) F (x) = $\frac{a^x - a^{-x}}{x}$, For x ≠ 0

= $(\log a)^2$, For x = 0 at x =0.

Answers :

(1)F(3) = 6 (2)F(0) = 0 (3)F(0) =e^4

(4)F(0) =3 (5)F(0) = $(\log 5)^2$ (6)F(0) = 2 log a.

(iv)F(x) = $\frac{\log(1+kx)}{\sin x}$, For x ≠ 0

= 5, For x = 0 at x = 0

(v)F(x) = $\frac{x\cos x+3\tan x}{x^2+\sin x}$, For x ≠ 0

=k^2, For x = 0 at x= 0.

(vi)F(x)= $\frac{(e^{kx}-1)\sin kx}{x^2}$, For x ≠ 0

= 4, For x = 0 at x = 0.

(vii)F(x) =$\frac{\cos 3x-\cos x}{x^2}$, For x ≠ 0

=k, For x = 0 at x = 0 **(Oct.05)**

(viii)F(x) = $\frac{8^x - 2^x}{k^x - 1}$, For x ≠ 0

=2, For x = 0 at x = 0.

2.Find F(0), if the function

F(x) = $\frac{\log(7+x)-\log(7-x)}{x}$, For x ≠ 0

Is continuous at x = 0 **(March'05)**

3. if the function f(x) = $\frac{(5^{\sin x}-1)^2}{x\log(1+2x)}$, x ≠ 0, is Continuous at x = 0, find f(0).

4. If the function F(x) =$\frac{(e^{3x}-1)\sin x}{x^2}$, x ≠ 0, is Continuous at x = 0, find F(0).

Answers:

(1)(i)±2 (ii)5 (iii)1/6 (iv)5 (v)±2 (vi)±2 (vii)-4 (viii)2.

(2)2/7 (3)$\frac{1}{2}(\log 5)^2$ (4)2/3 (5)3.

Assignment No:11

1.Find the value of k, if each of the following functions is continuous at the point shown against it:

(i)F(x) = $\frac{1-\cos kx}{x\sin x}$, For x ≠ 0

= 2, For x = 0 at x = 0. **(Oct.'01)**

(ii) F(x) =$(1+kx)^{1/x}$,For x ≠ 0

=e^5, For x = 0 at x = 0 **(Oct. '02)**

(iii)F(x) = $\frac{1-\sin x}{(\frac{\pi}{2}-x)^2}$, For x ≠ **π**/2

=3k, For x = **π**/2 at x= $\frac{\pi}{2}$.

Assignment No :12

Examine the following functions for continuity at the points shown against them:

(1)F(x) = 2x +- 3, For 0 ≤ x < 1

=3x + 2, For 1 ≤ x ≤2 at x = 1.

(2)F(x) = x^2 – x -1, For ≤ x < 2

=4x + 1, For 2 ≤ x < 4 at x = 2.

(3)F(x) = $x^2 + x + 1$, For $2 \le x \le 4$

=3x + 5, For $4 < x \le 6$ at x = 4.

(4)F(x) = $\frac{3x^2 - x - 2}{x-1}$, If $0 \le x < 1$

= 2x + 5, If $1 \le x \le 2$ at x =1.

(5)F(x) = 4 – 3x, For $0 < x \le 2$

= 2x – 6, For $2 < x \le 3$ at x= 2.

(6)F(x) = 2x – 6, For $2 < x \le 3$

= x + 5, For $3 < x \le 6$ at x = 3.

(7)F(x) = $x^2 + 1$, For $x \ge 0$

=2 $\sqrt{x^2 + 1}$ -1,For $x < 0$ at x = 0.

(8)F(x) = $\frac{x^2-16}{x-4}$, For $0 < x < 4$

= 4x + 1, For $4 \le x \le 8$ at x =4.

Answer :

Continuous: 1, 5, 7.

Discontinuous : 2, 3, 4, 6 ,8.

Assignment No : 13

A.Discuss the continuity of the following functions on the intervals shown below them of against them:

(1)F(x) = 4x – 7, For $x \neq 2$

F(2) = 3, On (-4, 4).

(2)F(x) = $\frac{x^2 + x - 12}{x^2 - 3x + 2}$, On [0, 4].

(3)F(x) = $\frac{x^2+1}{(x-1)(x-3)}$, On (-1, 2).

(4)F(x) = x^2 + x+ 3, If $0 \le x < 2$

=4x +1, If $2 \le x \le 4$ On [0, 4].

(5)F(x) = x^2 -4, For $2 \le x \le 4$

= 2x + 4, For $4 < x \le 6$ Over [2, 6].

(6)F(x) = x^2- 4, For $0 \le x \le 2$

=2x + 3, For $2 < x \le 4$ Over its

=x^2 – 5, For $4 < x \le 6$ domain.

B.Discuss the continuity of the following functions for all real values of x :

(1)F(x) = $\frac{x^3 - 2x + 1}{x^2 + 4}$

(2)F(x) = $\frac{3x+4}{x^2-1}$

(3)F(x) = $\frac{3x - 5\sin x}{x^2+2}$

(4)F(x) = $\frac{\sin x}{x^2 - 4}$

Answers :

A.(1)F is continuous on (-4, 4) except at x =2, where it is discontinuous.

(2) F is continuous on[0, 4] except at x = 1 and x=2, where it is discontinuous.

(3) F is continuous on (-1, 2) except at x=1. Note that $3 \notin (-1,2)$.

(4) F is continuous on $[0,4]$.

(5) F is continuous on $[2,6]$.

(6) F is continuous on its domain $[0,6]$ except at $x = 2$, where it is discontinuous.

B. 1. Continuous

2. Continuous except at $x = \pm 1$.

3. Continuous

4. Continuous except at $x = \pm 2$.

Assignment No – 14

1.Find the values of a and b, if the function F

(i) F(x) = $\frac{x^2 - 9}{x - 3} + a$ For $x > 3$

$= 5,$ For $x = 3$

$= 2x^2 + 3x + b,$ For $x < 3$

Is continuous at x= 3.

(ii) F(x) = ax + b, for $0 \le x \le 2$

=bx + 11, for $2 \le x \le 4$

Is continuous at x = 2 and F(3) =2.

(iii) F(x) = 2x – 3, for $x < 1$

= ax + b, for $1 \le x < 2$

= x^2 – 1, for $x \ge 2$

Is continuous at all x.

(iv) F(x) = x^2 + ax + b, for $0 \le x < 2$

= 4x – 1, for $2 \leq x \leq 4$

= $ax^2 + 17b$, for $4 < x \leq 6$

Is continuous on[0,6].

(v) F(x) = $\frac{\sin ax}{x} - 2$, for $-2 \leq x < 0$

= 2x + 1, for $0 \leq x \leq 1$

= 2b $\sqrt{x^2+3}$ - 1, for $1 < x \leq 2$

Is continuous on its domain.

2. If F is continuous at s = 0, where

F(x) = $\frac{\sin 3x}{5x} + a$, for $x < 0$

= x + 4 – b, for $x \geq 0$

Find the value of a + b. **(March '08)**

Answers :

1.(i) a = -1, b = -22

(ii)a = 4, b = -3

(iii)a = 4, b = -5

(iv)a = 2, b = -1

(v)a = 3, b = 1.

2. a + b = 17/5.

Assignment No – 15

1.Discuss the continuity of the function F given

By F(x) = $\frac{x^4-64x}{\sqrt{x^2+9}-5}$, for x ≠ 4

= 240, for x = 4 at x = 4

(March '04)

2. Test the continuity of the function F at x =0, if

F(x) = $\frac{(5^x-2^x)x}{\cos 5x-\cos 3x}$, for x ≠ 0

= $\frac{1}{8}\log(\frac{5}{2})$ for x = 0

3. Find the value of K, if the function F given by

F(x) = $\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$, for x ≠ **π**

= K, for x = π

Is continuous at x = **π**.

4. Find the value of K, if the function F given by

F(x) = $\log_{(1-2x)}(1+2x)$, for x ≠ 0

= K, for x = 0

Is continuous at x = 0.

5. Find the value of K, if the function

F(x) = $\frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}}$, for x ≠ 2

= K, for x = 2

Is continuous at x = 2.

6. If F(x)= $\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$, x ≠ **π**/2 is Continuous at x = **π**/2, find f(**π**/2).

7. If F(x) = $\frac{1+\cos \pi x}{(1-x)^2}$, x ≠ 1 is continuous at X = 1, find F(1).

8. Discuss the continuity on its domain, if

F(x) = $\frac{x^2-3x+2}{x-3}$, for $0 \leq x \leq 4$

= $\frac{x^2+1}{x-2}$, for $4 < x \leq 6$.

9. The function given by

F(x) = $\frac{x^2-4}{x-2}$ + a, for $x > 2$

= 3, for x = 2.

= $3x^2$ + 5x + b, for $x < 2$.

Is continuous at x = 2. Find a and b.

(March '02)

10. Find a and b if F is continuous at x = 1, where

F(x) = $\frac{\sin \pi x}{x-1} + a$, $x < 1$

= 2**π**, x = 1

= $\frac{1+\cos \pi x}{\pi(1-x)^2}$ + b, $x > 1$.

Answers :

1.Continuous 2. Discontinuous

3. K = ¼ 4. K = -1

5. K = -24 6. F(**π**/2) = $\frac{1}{4\sqrt{2}}$

7. F(1) = $\frac{\pi^2}{2}$

8. continuous on [0,6] except at x = 3 and x = 4.

9. a = -1, b= -19

10. a = 3**π**, b = $\frac{3\pi}{2}$.

Assignment No - 16

1.Find from first principles, the derivatives w.r.t X of the following functions:

(i) $\sqrt{x}$ (ii) $\frac{1}{3x+2}$ (iii) $\sqrt{x+3}$ (iv) $\frac{1}{\sqrt{x}}$

(v) $x^2+\sqrt{x+5}$ (vi) $\frac{1}{x\sqrt{x}}$ (vii) cos x

(viii) tan x **(March '02)**

(xi) sec x **(March '99)** (x) cos 2x **(Oct. '05)**

(xi) sin (2x + 3) (xii) $\sqrt{\sin x}$ (xiii) $\sin^3 x$

(xiv) x cos x **(March '04)** (xv) x tan x

(xvi) e^{7x-1} (xvii) a^{5x} (xviii) log x **(Oct. '03)**

(xix) log (3x -1) (xx) $\frac{1}{\log_a x}$.

2. Using definition, Find :

(i) F'(-1) if F(X) = $2x^2 + 4x - 5$

(ii) F'(2) if F(x) = $\sqrt{1+4x}$

(iii) F'(4) if F(x) = $\frac{3x+2}{x-1}$

(iv) F'(2) if F(x) = $\frac{1}{\sqrt{x+2}}$

(v) F'($\frac{\pi}{6}$) if F(x) = sin x.

Answers:

1.(i)$\frac{1}{2\sqrt{x}}$ (ii) $\frac{-3}{(3x+2)^2}$ (iii) $\frac{1}{2\sqrt{x+3}}$ (iv) $\frac{-1}{2x^{3/2}}$

(v)2x + $\frac{1}{2\sqrt{x+5}}$ (vi) $\frac{-3}{2x^{5/2}}$ (vii) – sin x

(vii) $\sec^2 x$ (ix) sec x tan x (x) -2 sin 2x

(xi) 2 cos (2x +3) (xii) $\frac{\cos x}{2\sqrt{\sin x}}$ (xiii) 3 $\sin^2 x$ cos x

(xiv) cos x – x sin x (xv) x $\sec^2 x$ + tan x

(xvi) $7e^{7x-1}$ (xvii) $5(a^{5x})$ log a (xviii) $\frac{1}{x}$

(xix) $\frac{3}{3x-1}$ (xx) $\frac{-\log a}{x(\log x)^2}$.

2. (i) 0 (ii) $\frac{2}{3}$ (iii) $\frac{-5}{9}$ (iv) $\frac{-1}{16}$ (v) $\frac{\sqrt{3}}{2}$.

Assignment No – 17

1.Differentiate the following w.r.t.x :

(i) $(3x^4 - x^3 + 4)^{5/2}$ (ii) $x+\sqrt{x^2+1}$

(iii) $\frac{1}{\sqrt{x+a}-\sqrt{x}}$ (iv) $\frac{1}{\sqrt{2x^2+5}+\sqrt{2x^2-3}}$

(v) log (sec x + tan x) (vi) log (log sin x)

(vii) $\log_5$ (log x) (viii) $\sin^5 4x + \tan^3 7x$

(ix) $\sqrt{a^2\cos^2 x + b^2\sin^2 x}$

(x) a^{4x} . cot 5x (xi) $(x^2+1)^5$ sin 4x

(xii) (2x + 1) $\sqrt{x^2+5}$ (xiii) $\sqrt{\frac{1-\cos x}{1+\cos x}}$

(xiv) log $\sqrt{\frac{1+\cos x}{1-\cos x}}$ (xv) $\frac{\sqrt{x^2+1}}{3+\sin 3x}$

(xvi) $\frac{\tan x}{e^{3x}+1}$

2. Find $\frac{dy}{dx}$, if

(i) y = log [e^x $(\frac{x-1}{x+2})^{1/2}$]

(ii) y = log [3^x $(\frac{x-4}{x+3})^{3/4}$]

(iii) y = log (x + $\sqrt{x^2+4}$)

(iv) y = $\log_{10}$ (x + $\sqrt{x^2+a^2}$)

(v) y = log ($e^{mx} - e^{-mx}$)

(vi) y = log ($\frac{x+\sqrt{x^2+25}}{\sqrt{x^2+25}-x}$

(vii) y = $7^{x \sin x}$

(viii) y = log [$\sin^3 x$. $\cos^4 x$. $(x^2-1)^5$]

3. If y = x + $\sqrt{a^2+x^2}$, show that $\frac{dy}{dx} = \frac{y}{\sqrt{a^2+x^2}}$

4. If y = t sin (2t + 5) + $\sqrt{1+t^2}$, find $\frac{dy}{dt}$

5.If y = log [$\frac{\sqrt{x+2}-2}{\sqrt{x+2}+2}$], show that

$\frac{dy}{dx} = \frac{2}{(x-2)\sqrt{x+2}}$

Answers :

1. (i)$\frac{15}{2}\ x^2(4x-1)(3x^4-x^3+4)^{3/2}$

(ii)$\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}$ (iii)$\frac{1}{2a}\left(\frac{1}{\sqrt{x+a}}+\frac{1}{\sqrt{x}}\right)$ **Hint :**Rationalise

(iv)$\frac{x}{4}\left[\frac{1}{\sqrt{2x^2+5}}-\frac{1}{\sqrt{2x^2-3}}\right]$ (v) sec x

(vi)$\frac{\cot x}{\log \sin x}$ (vii) $\frac{1}{x(\log 5)(\log x)}$

(viii)20 $\sin^4$4x cos4x + 21 $\tan^2$ 7x $\sec^2$7x

(ix) $\frac{(b^2-a^2)\sin x\cos x}{\sqrt{a^2\cos^2 x+b^2\sin^2 x}}$

(x) $a^{4x}[4(\log a)(\cot 5x)-5\text{cosec}^2 5x]$

(xi)2(x^2 + 1)^4 [2(x^2 + 1) cos4x + 5x sin 4x]

(xii)$\frac{4x^2+x+10}{\sqrt{x^2+5}}$ (xiii)$\frac{1}{2}\ sec^2\ \frac{x}{2}$

(xiv) – cosec x

(xv)$\frac{x(3+sin3x)-(3\cos 3x)(x^2+1)}{(3+\sin 3x)^2\ .\sqrt{x^2+1}}$

(xvi) $\frac{(e^{3x}+1).\ sec^2x-3e^{3x}.\tan x}{(e^{3x}+1)^2}$

2. (i) 1 + $\frac{1}{2(x-1)}-\frac{1}{2(x+2)}$

(ii) log 3 + $\frac{3}{4(x-4)}-\frac{3}{4(x+3)}$

(iii) $\frac{1}{\sqrt{x^2+4}}$ (iv) $\frac{1}{\sqrt{x^2+a^2}\ .\ \log 10}$

(v) $\frac{m(e^{mx}+e^{-mx})}{e^{mx}-e^{-mx}}$ (vi) $\frac{2}{\sqrt{x^2+25}}$

(vii) $7^{x\sin x}$ (log 7) (x cos x + sin x)

(viii) 3 cot x – 4 tan x + $\frac{10x}{x^2-1}$.

4.2t cos(2t + 5) + sin (2t + 5) + $\frac{t}{\sqrt{1+t^2}}$.

Assignment No – 18

Differentiate the following W. r. t.x:

1.(i)$\sin^{-1}(\frac{x}{a})$ (ii)$\cos^{-1}(\frac{x}{a})$

(iii) $\tan^{-1}(\frac{x}{a})$ (iv) $\csc^{-1}(\frac{x}{a})$

(v) $\sec^{-1}(\frac{x}{a})$ (vi) $\cot^{-1}(\frac{x}{a})$

2. (i)$\sin^{-1}(x+1)$ (ii)$\cos^{-1}(2-x)$

(iii)$\csc^{-1}(2x+1)$ (iv)$\tan^{-1}(3x^2+2)$

3.(i)$x^3\tan^{-1}x$ (ii)$(x^2+1)\sin^{-1}x$

(iii) $e^x\cot^{-1}x$ (iv) $\frac{\sin^{-1}x}{\sqrt{x}}$

(v) $\frac{\cos^{-1}x}{e^x}$

4.(i)$\cot^{-1}(\frac{1}{3x})$ (ii) $\tan^{-1}(\frac{1}{x})$

(iii) $\sec^{-1}(\frac{1}{x^2})$ (iv)$\csc^{-1}(\frac{1}{4x})$

(v) $\tan^{-1}$2x + $\tan^{-1}$ (1/2x).

5.(i)$\sin^{-1}(cos\ x)$ (ii)$\tan^{-1}(\cot 4x)$

(iii) $\cot^{-1}(\tan 3x)$ (iv)$\csc^{-1}(\sec 3x)$

(v)$\sec^{-1}(cos^{-1}\frac{7}{x})$

(vi) $\sin^{-1}(\cos x)+\cot^{-1}\left(\frac{1}{x}\right)+\sec^{-1}(\sqrt{2})$.

6.(i) $\sin^{-1}(2\ cos^2x-1)$ (ii)$\cos^{-1}2\ sin^2x$

(iii)$\cos^{-1}(\frac{2\tan x}{1-tan^2x})$

(iv)$\sin^{-1}(3sinx-4sin^3x)$

(v) $\cot^{-1}(4\ cos^3x-3\cos x)$.

7.(i)$\tan^{-1}(\frac{1-\cos x}{\sin x})$ (ii) $\tan^{-1}\sqrt{\frac{1+\cos 2x}{1-cos\ 2x}}$

(iii) $\cot^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ (iv) $\tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$

(v) $\tan^{-1}\sqrt{\frac{1-\cos 4x}{1+\cos 4x}}$ (vi) $\tan^{-1}(\frac{1+\sin x}{\cos x})$

(vii)$\tan^{-1}(\frac{\sin x}{1+\cos x})$ (viii) $\tan^{-1}(\frac{\cos x}{1-\sin x})$

8.(i)$\tan^{-1}(\frac{1-\tan 2x}{1+\tan 2x})$ (ii) $\tan^{-1}(\frac{\cos 5x+\sin 5x}{\cos 5x-\sin 5x})$

(iii) $\tan^{-1}(\frac{\cos 4x+\sin 4x}{\cos\ 4x-\sin 4x})$ **(Oct. '01)**

(iv)$\cot^{-1}(\frac{\cos 7x+\sin 7x}{\cos 7x-\sin 7x})$.

9.(i)$\cos^{-1}(\frac{3\cos x-2\sin x}{\sqrt{13}})$

(ii) $\sin^{-1}(\frac{\sqrt{3}\cos x+\sin x}{2})$

(iii) $\cos^{-1}[\frac{\sqrt{3}\cos(\frac{x}{2})+\sin(\frac{x}{2})}{2}]$

(iv) $\csc^{-1}(\frac{5}{4\sin x+3\cos x})$

10.(i) If y = x sin y, show that $\frac{dy}{dx}=\frac{sin^2 y}{\sin y-y\cos y}$.

(ii)If y = x cos y, show that $\frac{dy}{dx}=\frac{cos^2 y}{\cos y+y\sin y}$

(iii) If cos y = x cos (a+ y) , show that

$\frac{dy}{dx}=\frac{cos^2(a+y)}{\sin a}$.

Answers :

1.(i)$\frac{1}{\sqrt{a^2-x^2}}$ (ii) $\frac{-1}{\sqrt{a^2-x^2}}$ (iii)$\frac{a}{a^2+x^2}$

(iv)$\frac{-a}{|x\sqrt{x^2-a^2}|}$ (v)$\frac{a}{|x\sqrt{x^2-a^2}|}$

(vi)$\frac{-a}{a^2+x^2}$.

2.(i)$\frac{1}{\sqrt{1-(x+1)^2}}$ (ii)$\frac{1}{\sqrt{1-(2-x)^2}}$

(iii) $\frac{-1}{(2x+1)\sqrt{x^2+x}}$ (iv) $\frac{6x}{1+(3x^2+2)^2}$

3.(i)$\frac{x^3}{1+x^2}+3x^2\tan^{-1}x$

(ii) $\frac{x^2+1}{\sqrt{1-x^2}}+2x\ \sin^{-1}x$

(iii) $e^x(\cot^{-1}x-\frac{1}{1+x^2})$

(iv) $\frac{2x-\sqrt{1-x^2}(\sin^{-1}x)}{2x^{3/2}\sqrt{1-x^2}}$

(v) $\frac{-\sqrt{1-x^2}\,(\cos^{-1}x)-1}{e^x\sqrt{1-x^2}}$

4.(i)$\frac{3}{1+9x^2}$ (ii) $\frac{-1}{1+x^2}$ (iii) $\frac{-2x}{\sqrt{1-x^4}}$

(iv)$\frac{4}{\sqrt{1-16x^2}}$

(v)0. **Hint :** use $\tan^{-1}2x+\cot^{-1}2x=\pi/2$

5.(i)-1 (ii) -4 (iii) -3 (iv) -3 (v) 1/7 (vi)$\frac{-x^2}{1+x^2}$

6.(i)-2 (ii) 2 (iii) -2 (iv) 3 (v) 3.

7.(i)1/2 (ii)-1 (iii) -1/2 (iv) -1/2 (v) 2 (vi) ½

(vii) ½ (viii) 1/2.

8.(i) -2 (ii) 5 (iii) 4 (iv) -7.

9. (i) 1 (ii) 1 (iii) ½ (iv) 1.

Assignment No – 19

Differentiate the following w.r.t.x :

1.$\sin^{-1}\sqrt{1-x^2}$ 2.$\tan^{-1}(\frac{x}{\sqrt{1-x^2}})$

3. $\cot^{-1}(\frac{\sqrt{1-x^2}}{x})$ 4.$\csc^{-1}\sqrt{1+x^2}$

5. $\sec^{-1}x+\csc^{-1}\frac{x}{\sqrt{x^2-1}}$

6. $\sin^{-1}(2x\sqrt{1-x^2})$

7.$\cos^{-1}(1-x^2)$ 8.$\cot^{-1}(\frac{1-x^2}{2x})$

9.$\tan^{-1}(\frac{1-x^2}{2x})$ 10.$\csc^{-1}(\frac{1}{2x\sqrt{1-x^2}})$

(March '04)

11.$\sin^{-1}(3x-4x^3)$ 12.$\cos^{-1}(4x^3-3x)$

13.$\tan^{-1}(\frac{3x-x^3}{1-3x^2})$ 14.$\sin^{-1}(4x^3-3x)$

15.$\sin^{-1}(\frac{2x}{1+x^2})$ 16.$\cos^{-1}(\frac{1-x^2}{1+x^2})$

(Oct. '96)

17.$\sin^{-1}(\frac{2x^2}{1+x^4})$ 18. $\sin^{-1}(\frac{1-x^2}{1+x^2})$

19.$\sec^{-1}(\frac{x^2+1}{x^2-1})$ 20.$\sin^{-1}(\frac{6x}{1+9x^2})$

21.$\cos^{-1}(\frac{1-16x^2}{1+16x^2})$ 22.$\sin^{-1}(\frac{1-x}{1+x})$

23.$\text{cosec}^{-1}(\frac{1+25x^2}{10x})$ 24.$\sec^{-1}(\frac{1+x}{2\sqrt{x}})$

25.$\cot^{-1}(\frac{1-x}{2\sqrt{x}})$ 26. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

27.$\tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$ 28.$\tan^{-1}(x+\sqrt{1+x^2})$

29.$\tan^{-1}(\frac{1+\sqrt{1-x^2}}{x})$ 30.$\tan^{-1}(\frac{x}{1-\sqrt{1-x^2}})$.

Answers: (1)$\frac{-1}{\sqrt{1-x^2}}$. Put x = cos θ

(2)$\frac{1}{\sqrt{1-x^2}}$. Put x = sinθ

(3) $\frac{1}{\sqrt{1-x^2}}$. Put x = sin θ

(4) $\frac{-1}{1+x^2}$. Put x = cot θ

(5) $\frac{2}{|x\sqrt{x^2-1}|}$. Put x = sec θ

(7) $\frac{2}{\sqrt{1-x^2}}$. Put x = sin θ

(8) $\frac{2}{1+x^2}$. Put x = tan θ

(9) $\frac{-2}{1+x^2}$. Put x = tan θ

(10) $\frac{2}{\sqrt{1-x^2}}$. Put x = sin θ

(11) $\frac{3}{\sqrt{1-x^2}}$. Put x = sin θ

(12) $\frac{-3}{\sqrt{1-x^2}}$ .Put x = cos θ

(13) $\frac{3}{1+x^2}$. Put x = tan θ

(14) $\frac{3}{\sqrt{1-x^2}}$. Put x = cos θ

(15) $\frac{2}{1+x^2}$. Put x = tan θ

(16) $\frac{2}{1+x^2}$. put x = tan θ

(17) $\frac{4x}{1+x^4}$. Put x^2 = tanθ

(18) $\frac{-2}{1+x^2}$. Put x = tan θ

(19) $\frac{-2}{1+x^2}$. Put x = tan θ

(20) $\frac{6}{1+9x^2}$. Put 3x = tan θ

(21) $\frac{8}{1+16x^2}$. Put 4x = tan θ

(22) $\frac{-1}{\sqrt{x}(1+x)}$. Put $\sqrt{x}$ = tan θ

(23) $\frac{10}{1+25x^2}$. Put 5x = tan θ

(24) $\frac{-1}{\sqrt{x}(1+x)}$. Put $\sqrt{x}$ = tan θ

(25) $\frac{1}{\sqrt{x}\ (1+x)}$. Put $\sqrt{x}$ = tan θ

(26) $\frac{-1}{2\sqrt{1-x^2}}$. Put x = cos θ

(27) $\frac{1}{2(1+x^2)}$. Put x = tan θ

(28) $\frac{1}{2(1+x^2)}$. Put x = tan θ

(29) $\frac{-1}{2\sqrt{1-x^2}}$. Put x = sin θ

(30) $\frac{-1}{2\sqrt{1-x^2}}$. Put x = sin θ.

Assignment No-20

Differentiate the following w.r.t.x :

1.$\tan^{-1}(\frac{2-3x}{1+6x})$ 2.$\tan^{-1}(\frac{1-x^2}{1+x^2})$

3.$\tan^{-1}(\frac{8x}{1-16x^2})$ 4.$\tan^{-1}(\frac{7x}{1-12x^2})$

5.$\tan^{-1}(\frac{x-\sqrt{x}}{1+\sqrt{x^3}})$ 6.$\tan^{-1}(\frac{7x}{1-12x^2})$

7.$\tan^{-1}(\frac{2+3x}{3-2x})$ 8.$\cot^{-1}(\frac{2-5x}{5+2x})$

9.$\tan^{-1}(\frac{ax+b}{a-bx})$ 10.$\cot^{-1}(\frac{ax+b}{a-bx})$

11.$\tan^{-1}(\frac{2+3\tan x}{3-2\tan x})$ 12.$\cot^{-1}(\frac{3-2\tan x}{2+3\tan x})$

13.$\tan^{-1}(\frac{a+b\cos x}{b-a\cos x})$ 14.$\tan^{-1}(\frac{6+5\tan x}{5-6\tan x})$

15.$\cot^{-1}(\frac{x-\sin x}{1+x\sin x})$

Answers : (1) $\frac{-3}{1+9x^2}$ (2) $\frac{-2x}{1+x^4}$ (3) $\frac{8}{1+16x^2}$

(4) $\frac{3}{1+9x^2}$ (5) $\frac{2}{1+4x^2}$ (6) $\frac{1}{1+x^2}$ + $\frac{3}{1+x^2}$

(7) $\frac{1}{1+x^2}$ (8) $\frac{1}{1+x^2}$ (9) $\frac{1}{1+x^2}$ (10) $\frac{-1}{1+x^2}$

(11)1 (12) 1 (13) $\frac{-\sin x}{1+\cos^2 x}$ (14)1

(15) $\frac{\cos x}{1+\sin^2 x}$ - $\frac{1}{1+x^2}$.

Assignment No-21

1.Using first principles find the derivatives w.r.t.x of the following:

(i)$5x^2$- $\sqrt{x^2+2+3}$ (ii)$x^2\sin x$

(iii)$\sin 2x + \sqrt{\tan x}$ (iv)$\log_a(x+2)$

(v)$\log(\sqrt{2x-3})$

2. show that the function whose graph is a right Angle is not differentiable at its vertex.

(March. '97)

[Hint : consider the function F(x) = |x| It is given as for x ≥ 0,F(x) = x, i.e, y = x

For x < 0,F(x) = -x, i.e., y =-x

Its graph is shown in the diagram.

It is a right angle whose vertex is at the origin.

We show that F is not differentiable at x = 0. For this we have to show that

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

Does not exist. This is proved in the second part of theorem 3.1.]

3. If y = $(x^2+1)^3\sqrt{x^2+4}$, find $\frac{dy}{dx}$

4. If y = log($\frac{1-\cos x}{1+\cos x}$),

show that $\frac{dy}{dx}$ = 2 cosec x.

5. If y = x + $\sqrt{1-x^2}$, show that $\frac{dy}{dx}=\frac{y}{y-x}$

6. If y = sin ($2\sin^{-1}x$),

show that $\frac{dy}{dx}=2\sqrt{\frac{1-y^2}{1-x^2}}$.

7. Find $\frac{dy}{dx}$, if(i

(i)Y = $\tan^{-1}(\sec x+\tan x)^2$

(ii)y = $\tan^{-1}(-x+\sqrt{1+x^2})$

8. If y = $\cot^{-1}[\frac{x-\log x^{x^2}}{\log e^{x^2}+\log x^x}]$, find $\frac{dy}{dx}$

9. Find $\frac{dy}{dx}$, if

(i)y = $\tan^{-1}(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}})$

(ii)y = $\cos^{-1}(\frac{8x}{1+16x^2})$.

10. Find $\frac{dy}{dx}$, if

(i)y =tan-1 ($\frac{\sqrt{x}}{1+20x}$)

(ii)y = cot-1[$\frac{-(x^2+7x+11)}{2x+7}$]

(iii)y = sin-1 ($\frac{\sqrt{1+sinx}+\sqrt{1-\sin x}}{\sqrt{1+sin\,x}-\sqrt{1-\sin x}}$)

11.If y = cot-1 [$\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$] ,

Show that $\frac{dy}{dx}$ = 1/2

12.If y = $\text{cosec}^{-1}[\frac{\sqrt{13}}{2x+3\sqrt{1-x^2}}]$, show

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$.

Answers:

1.(i)10x - $\frac{x}{\sqrt{x^2+2}}$

(ii)x^2cos x + 2x sin x

(iii)2cos 2x + $\frac{sin^2 x}{2\sqrt{tanx}}$

(iii)2cos 2x+ $\frac{sec^2 x}{2\sqrt{\tan x}}$

(iv) $\frac{1}{(x+2)\log a}$

(v) $\frac{1}{2x-3}$

3. $\frac{x(x^2+1)^2(7x^2+25)}{\sqrt{x^2+4}}$

7. (i)$\frac{\cos x}{1+\sin^2 x}$ **(ii)**$\frac{-1}{2(1+x^2)}$

8. $\frac{1}{1+x^2}+\frac{1}{x[1+(log\,x)^2]}$

9. (i)$\frac{1}{2\sqrt{1-x^2}}$ **(ii)**$\frac{-8}{1+16x^2}$

10. (i)$\frac{1}{2\sqrt{x}}[\frac{5}{1+25x}-\frac{4}{1+16x}]$

(ii) $\frac{1}{x^2+6x+10}+\frac{1}{x^2+8x+17}$

[Hint : $\frac{-(x^2+7x+11)}{2x+7}=\frac{1-(x^2+7x+12)}{2x+7}=\frac{1-(x+3)(x+4)}{(x+3)(x+4)}$ **]**